

# OPTIMIZED RECEIVE FILTERS AND PHASE-CODED PULSE SEQUENCES FOR CONTRAST AGENT AND NONLINEAR IMAGING

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*Abstract* – Phase- or amplitude-coded pulse sequences are commonly used to discriminate linear from nonlinear scatterers or to enhance image quality by imaging spectral components that are due to nonlinear sound propagation [1]. The transmit pulse sequences are designed so that a coherent, weighted summation of the resultant echoes followed by a demodulation enhances the image contrast between nonlinear and linear scatters or the separation of higher harmonics from the fundamental. Because of limitations in the signal generation in ultrasound scanners and unknown effects of the sound propagation, experimental results do not agree with those of simulations. Separate receive filters for each of the echoes can compensate for the above-mentioned shortcomings and unknown factors. In the following, an algorithm for the design of FIR filters based on training data is presented and experimental results are discussed.

## INTRODUCTION

Phase-coded pulse sequences can be used to distinguish nonlinear from linear scatterers or media. To form one line of an image,  $N$  echoes are acquired, where each echo results from a different transmit pulse. The pulses have the same envelope and carrier frequency but different carrier phases and/or amplitudes. A weighted summation of the echoes can cancel out the part of the signal that results from linear scattering and propagation, i. e. the fundamental. Depending on the carrier phases used, harmonics or sub-harmonics are enhanced.

Commercial scanners reproduce the desired transmit pulses inaccurately. Thus, the suppression of the fundamental is incomplete. This problem can be solved by linear receive filters, where  $N$  different filters are assigned to the  $N$  transmit pulses.

Design criteria are an optimal suppression of the fundamental or optimal echo energy ratio between

two media (contrast agent / tissue) after summation. Due to various nonlinear effects, the 2 criteria are not identical, and the latter is of greater practical interest. Since convolution and summation are linear operations, the problem can be described and solved by linear algebra.

## THEORY

The transmit signals can be written as

$$s_{i0}(t) = a_i \cdot g(t) \cdot \cos(\omega_0 t + \varphi_i), \quad i = 1 \dots N, \quad (1)$$

$a_i$  (amplitude),  $\omega_0$  (carrier freq.),  $\varphi_i$  (phase)  $\in \mathbb{R}$ .

Due to system-inherent errors, a more general formulation might be necessary:

$$s_{i0}(t) = a_i \cdot g_i(t) \cdot \cos(\omega_i t + \varphi_i), \quad i = 1 \dots N, \quad (2)$$

$a_i, \omega_i, \varphi_i \in \mathbb{R}, \quad g_i(t) \approx g(t), \quad \omega_i \approx \omega_0$

To allow for an echo to have returned from the maximum imaging depth before the next pulse of the sequence is transmitted, a delay time  $T_{\text{PRI}}$  is introduced between the pulses. Hence, the complete transmit pulse sequence is given by

$$s_0(t) = \sum_{i=1}^N s_{i0}(t - (i-1) \cdot T_{\text{PRI}}). \quad (3)$$

All non-linearities on the transmit side are included in  $s_0$ . A linear system on the transmit side, e. g. the frequency response of the transducer, may be described by a linear impulse response  $h_{0T}$ . Considering  $h_{0T}$ , we form a new transmit signal

$$s(t) = \sum_{i=1}^N s_i(t - (i-1) \cdot T_{\text{PRI}}), \quad (4)$$

$$s_i(t) = s_{i0}(t) * h_{0T}(t).$$

The echo signals will reflect the delay time  $T_{\text{PRI}}$ :

$$e_0(t) = \sum_{i=1}^N e_{i0}(t - (i-1) \cdot T_{\text{PRI}}). \quad (5)$$

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The impulse response of a linear system on the receive side can be included by  $h_{0R}$ :

$$\begin{aligned} e(t) &= \sum_{i=1}^N e_i(t - (i-1) \cdot T_{\text{PRI}}), \\ e_i(t) &= e_{i0}(t) * h_{0R}(t) \end{aligned} \quad (6)$$

After time gating, we readjust the time axis for the echoes so that the echoes appear to be simultaneous regardless of  $T_{\text{PRI}}$ .

The propagation and reflection of a linear medium is characterized by an impulse response  $q(t)$ .

$$e_{i0}(t) = s_i(t) * q(t) \quad (7)$$

The processing of the echoes consists of a convolution, where  $N$  different filters are assigned to the  $N$  echoes, and a summation of the filtered echoes:

$$r(t) = \sum_{i=1}^N e_i(t) * f_i(t) \quad (8)$$

If the pulses fulfill (1) and have different phases or amplitudes, filters with a constant, non-zero frequency response can be found so that

$$r(t) = 0 \quad \forall \quad t. \quad (9)$$

For the more practical case (2), the filters will have to limit the bandwidth to fulfill (9). For a nonlinear medium,  $r(t)$  is generally non-zero. This fact is used to detect nonlinear scatterers and nonlinear propagation. For contrast agent imaging or nonlinear tissue imaging, two different media have to be discriminated from one another. Both media will have to be described by nonlinear impulse responses  $q_1(s(t))$  and  $q_2(s(t))$ .

For any location, the impulse responses will not only describe the medium at that location (scattering) but also the medium between the transducer and the location (propagation), which in general is unknown. For contrast agent imaging, we assume that nonlinear scattering dominates over nonlinear propagation. We can, therefore, simplify the problem by comparing the two media within the same depth range. The transmit path through any medium – linear or nonlinear – between the transducer and a considered depth range results in a modified excitation signal for this depth. Nonlinear propagation on the receive path is negligible due to the low amplitudes after the scattering, and linear propagation may be included in  $h_{0R}(t)$ , see (6).

To enhance the image contrast between two media  $M_1$  and  $M_2$  by an optimized set of filters  $f_i$ ,  $r(t)$  has to be analyzed for both media. Hence, we denote

$$\begin{aligned} {}^1r(t) &= \sum_{i=1}^N {}^1e_i(t) * f_i(t) \quad \text{receive signal, } M_1, \\ {}^2r(t) &= \sum_{i=1}^N {}^2e_i(t) * f_i(t) \quad \text{receive signal, } M_2. \end{aligned} \quad (10)$$

The energy ratio of the two receive signals provides a measure of the image contrast:

$$c = \frac{\int_t \left[ \sum_{i=1}^N {}^1e_i(t) * f_i(t) \right]^2 dt}{\int_t \left[ \sum_{i=1}^N {}^2e_i(t) * f_i(t) \right]^2 dt} \quad (11)$$

A single beam line is not representative for the acoustic properties of a medium. Thus, we acquire several beam lines for both media, i. e.  $K_1$  lines for the medium  $M_1$ , and  $K_2$  lines for the medium  $M_2$ . The contrast is then given by

$$c = \frac{\frac{1}{K_1} \sum_{k=1}^{K_1} \int_t \left[ \sum_{i=1}^N {}^1e_{ki}(t) * f_i(t) \right]^2 dt}{\frac{1}{K_2} \sum_{k=1}^{K_2} \int_t \left[ \sum_{i=1}^N {}^2e_{ki}(t) * f_i(t) \right]^2 dt} \quad (12)$$

For the conversion of the problem into the discrete-time domain we denote:

$$t = l \cdot T, l \in \mathbb{Z}, T \in \mathbb{R}^+, \quad (13)$$

where  $T$  is the sampling interval. All  $K$  beam lines that correspond to a medium  $M$  shall cover the same depth range in the time range  $L \cdot T$ . The index  $l$  for the minimal depth is defined as 0. The length of the filters is set to  $J$  taps. A convolution of a signal with  $L$  samples with a  $J$ -tap filter yields a signal with  $L + J - 1$  samples. Hence:

$$c = \frac{\frac{1}{K_1} \sum_{k=1}^{K_1} \sum_{l=0}^{L+J-2} \left[ \sum_{i=1}^N {}^1e_{ki}(l \cdot T) * f_i(l \cdot T) \right]^2}{\frac{1}{K_2} \sum_{k=1}^{K_2} \sum_{l=0}^{L+J-2} \left[ \sum_{i=1}^N {}^2e_{ki}(l \cdot T) * f_i(l \cdot T) \right]^2}, \quad (14)$$

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where in any expression  $[s(l \cdot T)]^2$  the samples, i. e. the vector components, are squared.

The convolution in (14) can be written as a multiplication of a matrix with a vector [2]:

$$e_i(l \cdot T) * f_i(l \cdot T) = \mathbf{E}_i \cdot f_i, \quad (15)$$

$$f_i = (f_{i,0} \quad f_{i,1} \quad \cdots \quad f_{i,J-1})^T, f_{i,l} = f_i(l \cdot T)$$

With (15), a simplified formulation of the summation in (8) is

$$\sum_{i=1}^N e_i(l \cdot T) * f_i(l \cdot T) = \mathbf{E} \cdot f, \quad (16)$$

$$\mathbf{E} = [\mathbf{E}_1 \quad \mathbf{E}_2 \quad \cdots \quad \mathbf{E}_N],$$

$$f = (f_1^T \quad f_2^T \quad \cdots \quad f_N^T)^T.$$

The energy of a time-discrete receive signal  $r(l \cdot T)$  is expressed in

$$|r|^2 = T \cdot \sum_{l=0}^{L+J-2} \left[ \sum_{i=1}^N e_i(l \cdot T) * f_i(l \cdot T) \right]^2$$

$$= T \cdot f^T \cdot \mathbf{E}^T \cdot \mathbf{E} \cdot f, \quad (17)$$

$$r = (r_0 \quad r_1 \quad \cdots \quad r_{L+J-2})^T, r_l = r(l \cdot T).$$

The average energy resulting from  $K$  beam lines equals

$$\frac{T}{K} \cdot \sum_{k=1}^K \sum_{l=0}^{L+J-2} \left[ \sum_{i=1}^N e_i(l \cdot T) * f_i(l \cdot T) \right]^2$$

$$= \frac{T}{K} \cdot \sum_{k=1}^K f^T \cdot {}_k \mathbf{E}^T \cdot {}_k \mathbf{E} \cdot f = T \cdot f^T \cdot \mathbf{E}' \cdot f, \quad (18)$$

$$\mathbf{E}' = \frac{1}{K} \sum_{k=1}^K \mathbf{E}^T \cdot {}_k \mathbf{E}.$$

Hence, (14) can be rewritten:

$$c = \frac{f^T \cdot {}^1 \mathbf{E}' \cdot f}{f^T \cdot {}^2 \mathbf{E}' \cdot f} \quad (19)$$

Our aim is to optimize the contrast  $c$  as a function of the  $N$  filters represented in  $f$ . The straightforward approach, i. e. calculation the first derivative of  $c$ , would lead to a nonlinear equation system with  $N \cdot J$  equations. Instead, we formulate the problem as an optimization problem with the constraint that the filters in  $f$  fulfill the following condition

$$f^T \cdot {}^2 \mathbf{E}' \cdot f = 1. \quad (20)$$

Since the problem given in (19) is invariant with respect to a scaling of  $f$ , the normalization (20) is possible and indispensable. Combining (19) and (20) yields

$$c = f^T \cdot {}^1 \mathbf{E}' \cdot f \quad (21)$$

We will now determine the filters  $f$  that maximize  $c$  with the constraint (20). The optimization problem can then be solved by means of Lagrange multipliers [3]. The function that has to be optimized is

$$f^T \cdot {}^1 \mathbf{E}' \cdot f + \lambda \cdot (f^T \cdot {}^2 \mathbf{E}' \cdot f - 1). \quad (22)$$

Using the derivative of (22), the solution for  $f$  can be found by the following equation system:

$${}^1 \mathbf{E}' \cdot f + \lambda \cdot {}^2 \mathbf{E}' \cdot f = 0 \quad (23)$$

(23) represents a generalized Eigenvalue problem. Since  ${}^2 \mathbf{E}'$  is invertible, a left multiplication of the equation (23) by the inverse of  ${}^2 \mathbf{E}'$  leads to the traditional Eigenvalue problem

$$({}^2 \mathbf{E}')^{-1} \cdot {}^1 \mathbf{E}' \cdot f + \lambda \cdot f = 0 \quad (24)$$

The Eigenvectors contain filter coefficients of all  $N$  filters. After scaling the Eigenvectors to fulfill (20), the Eigenvector that maximizes the contrast  $c$  has to be determined by evaluating (21).

## EXPERIMENTAL RESULTS

To acquire in vitro data, a tissue-mimicking phantom with a cylindrical hole containing Levovist was imaged with a Siemens Sonoline<sup>®</sup> Elegra. The hole has a diameter of 2.5 cm and is positioned at an imaging depth of 5 cm.

### 4-pulse sequence at 2.0 MHz

The first experiment was conducted using a 3.5 MHz curved array, where the pulse sequence consisted of 4 pulses with  $\varphi_i = [0^\circ, 120^\circ, 180^\circ, 240^\circ]$  and with a carrier frequency of  $\omega_0 = 2.0$  MHz.

The contrast agent filled space can easily be identified in Figure 1 as an irregularly filled circular region that causes shadowing because of the fairly high concentration of the contrast agent Definity<sup>™</sup>. To further analyze the discrimination between contrast agent and tissue, normalized histograms for the 2 media were calculated.

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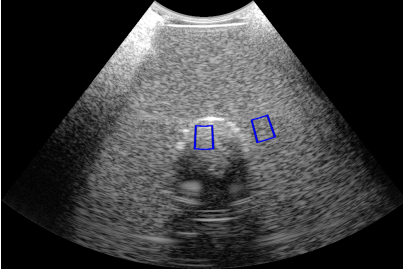


Figure 1: B mode image of the contrast agent phantom. Dynamic range: 55 dB. Boxes: sample regions for filter optimization (left: contrast agent, right: tissue).

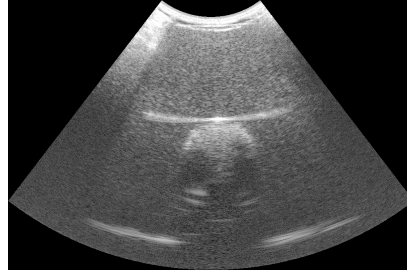


Figure 3: Demodulated image after optimized 1-tap filtering. Dynamic range: 55 dB.

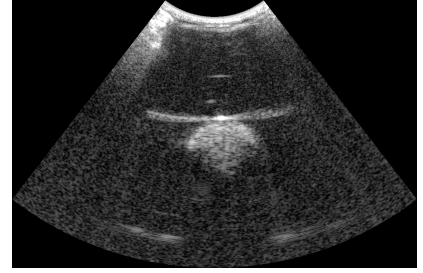


Figure 5: Demodulated image after optimized 64-tap filtering. Dynamic range: 55 dB.

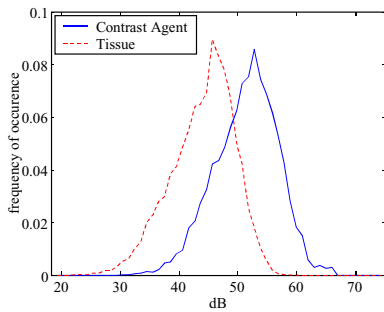


Figure 2: Normalized histograms for the representation of contrast agent and tissue in the image shown in Figure 1.

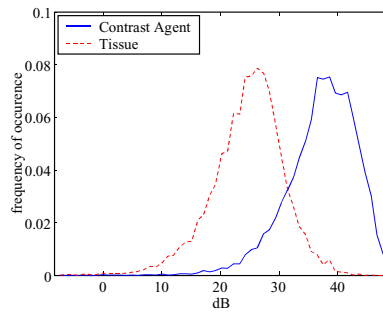


Figure 4: Normalized histograms for the representation of contrast agent and tissue in the image shown in Figure 3.

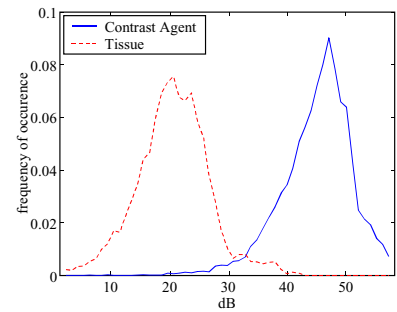


Figure 6: Normalized histograms for the representation of contrast agent and tissue in the image shown in Figure 5.

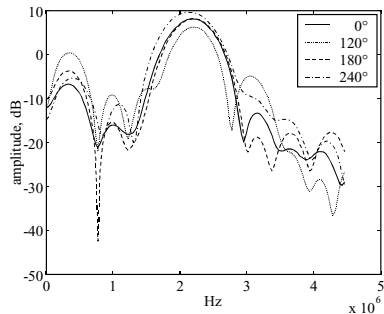


Figure 7: Amplitude spectra of the 64-tap filters.

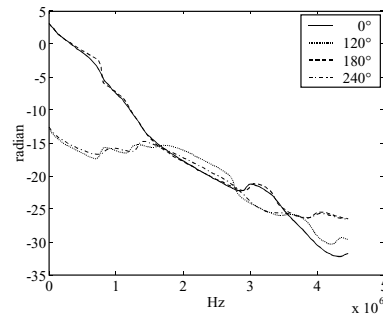


Figure 8: Phase spectra of the 64-tap filters.

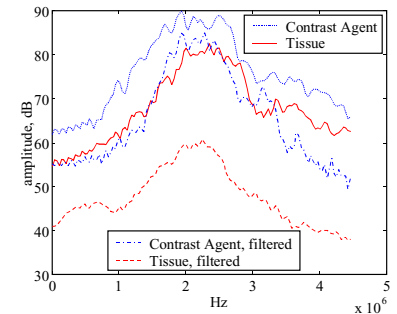


Figure 9: Amplitude spectra for tissue and contrast agent before and after filtering and summation.

The histograms in Figure 2 show a significant overlap, which is determined by the reflectivity of the tissue and the reflectivity, i. e. the concentration, of the contrast agent.

A coherent weighted summation of the 4 echoes per beam line, where the weights are optimized with respect to (12), leads to the image shown in Figure 3. The weights represent a set of 4 1-tap filters. The corresponding histograms, see Figure 4, show a re-

duced but still substantial overlap. The poor result is due to the fact that a 1-tap filter cannot correct for broadband, frequency dependent amplitude and phase errors. The weighted summation partly suppresses the fundamental and, therefore, most of the echo energy, so that the noise floor becomes evident in the image.

The improvement achieved by extending the filter length to 64 taps is illustrated in Figure 5. The visual

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impression is confirmed by the histograms given in Figure 6. The histograms confirm an almost complete separability of the 2 media. Setting an optimal threshold at 33 dB results in a total classification error of less than 3.5 % in the depth range of the sample regions (B mode, threshold: 48 dB, error: 24.5 %).

To better understand the filtering process, the amplitude and phase spectra of the filters are shown in Figure 7 and Figure 8, respectively. The filters for the echoes corresponding to the  $0^\circ$  and  $180^\circ$  carrier phase are very similar, since a bipolar transmitter can achieve a phase shift of  $180^\circ$  most accurately, while other phase shifts require more advanced pulse shaping capabilities than most commercial scanners offer. Thus, the filters for the  $120^\circ$ - and the  $240^\circ$  differ from the other two significantly and do not show a perfect symmetry. This is especially noticeable in the phase spectra. The filters correct for errors in amplitude and phase by introducing a frequency dependent amplitude weighting and phase shift. Furthermore, the sub-harmonic frequency range and the frequency range that matches the transducer bandwidth best are favored while others are suppressed. The preferred frequency ranges are those that enhance the contrast between the media. It is important to note that these frequency ranges are not necessarily those that should be used for single transmit harmonic imaging, because enhancement and suppression of harmonics is predominantly achieved by the summation due to the phase relationships within the multi-pulse sequence. It is interesting to note that the original echoes contain significant energy in the frequency range of 3 – 5 MHz. The filters suppresses this frequency range, see Figure 9, indicating that this part of the spectrum does not allow the discrimination of the 2 media.

#### *5-pulse sequence at 3.6 MHz*

Another experiment was conducted with Levovist<sup>®</sup> using a 7.2 MHz linear array, where the pulse sequence consisted of 5 pulses with  $\varphi_i = (i-1) \cdot 72^\circ$  and with a carrier frequency of  $\omega_0 = 6$  MHz. In this case, the separation of the 2 media less than 1 %. 10 different subsets of 3 out of 5 echoes per beam line were processed to form 10 demodulated A-lines. These A-lines were then averaged to give one line of the image. This procedure, but with a simple weighted summation instead of a summation after

optimal filtering, was proposed in [4,5]. Alternatively, all 5 echoes were filtered, summed and demodulated to form a line of an image. In the former case, the resultant image showed less speckle noise. In the latter case, the axial resolution was slightly better. The spatial resolution was very poor in both cases. Further analysis revealed that the filters limited the frequency range to sub-harmonics (0 – 2 MHz). Due to the broadband excitation of the transducer, the transmitted spectrum showed a center frequency of approximately 5 MHz. This frequency is higher than the resonant frequency of the insonified microbubbles. Consequently, the generation of higher harmonics is unlikely. Further experiments will be conducted to explore the use of 5-pulse sequences at lower frequencies.

#### CONCLUSION

The proposed optimal receive filters have the potential to greatly improve nonlinear contrast agent imaging using multi-pulse sequences. Phase and amplitude errors on the transmit side of ultrasound scanners can be compensated by the optimal receive filters.

Since the optimization process is not only applicable to contrast agent and tissue but to any two media that differ in terms of non-linearity or frequency dependent backscattering or attenuation, the same approach can be used to differentiate between different types of tissue.

#### REFERENCES

- [1] C. Chapman, J. C. Lazenby, "Ultrasound imaging system employing phase inversion subtraction to enhance the image," US patent 5,632,277, 1997.
- [2] K. R. Griep, J. A. Ritcey, J. J. Burlingame, "Poly-Phase Codes and Optimal Filters for Multiple User Ranging," IEEE Transactions on Aerospace and Electronic Systems, Vol. 31, No. 2, Apr. 1995.
- [3] Handbook of Mathematics by I. N. Bronstein, K.A. Semendyayev.
- [4] H. Ermert, J. Lazenby, M. Krueger, C. Chapman, W. Wilkening, "Diagnostic ultrasonic imaging system and method for discriminating non-linearities," US patent 6,155,981, 2000.
- [5] W. Wilkening, M. Krueger, H. Ermert, "Phase-Coded Pulse Sequence for Non-Linear Imaging," Proceedings of the IEEE Ultrasonics Symposium, 2000, 1D-2.

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